# Lecture #4: Analog-to-Digital Conversion

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**Most slides obtained from 12-740 for Fall 2006*

Civil & Environmental ENGINEERING
Carnegie Mellon

# But first…

- How are you holding up so far?
- Would you like me to set up a Piazza site?
- Have you thought about your projects?
- Office hours

# Recap

- Sensors, Actuators, Transducers?
- What is Sensor Andrew?
- Transfer Function, Accuracy, Sensitivity
- Why can sensors have dynamic characteristics?
- Why do we need to condition signals?
- How are the dynamic characteristics of an electrical circuit relevant to this class?

# Agenda for today

- Finish basics of circuit analysis
- Start discussing about the Analog-to-Digital-Conversion

# PEN AND PAPER TIME…

# Recap

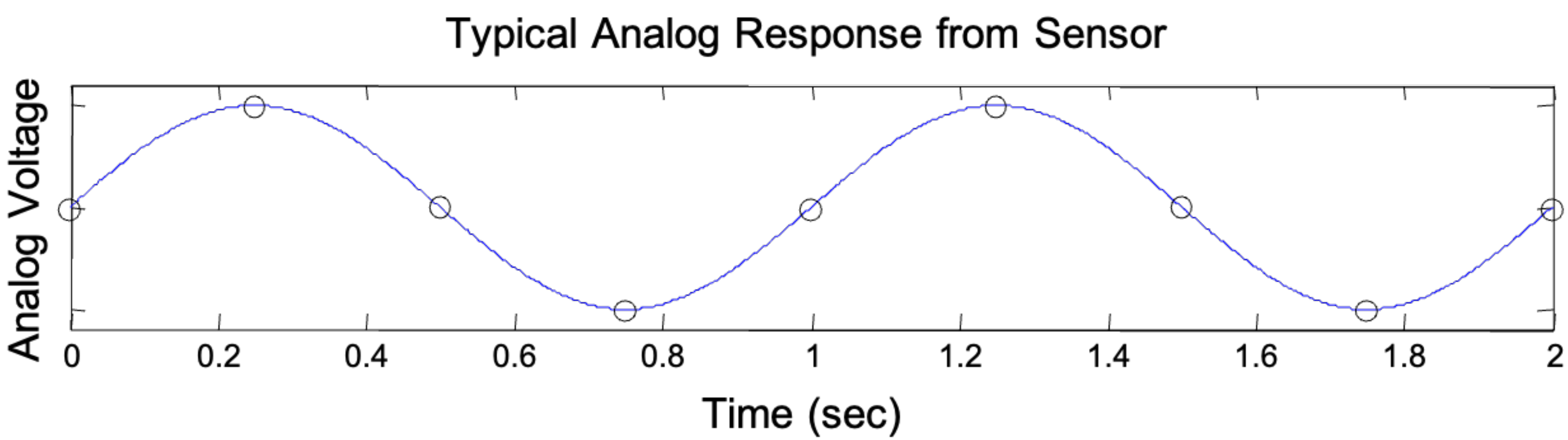
- We reviewed basics of electrical circuits.
  - Are the concepts fresh now?
- We did not finish, so let’s do that now:
  - Frequency response functions
  - Voltage dividers
    - Low-pass filters
    - High-pass filters

Dally (Chapter2) and others

# SLIDES BASED ON

# Digitization of Analog Signals

- Sampling: We need to specify how often the data is sampled in time axis.
- Quantization: The amplitude of the analog sensor reading is digitized by A/D converter.

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# Determination of Sampling Frequency

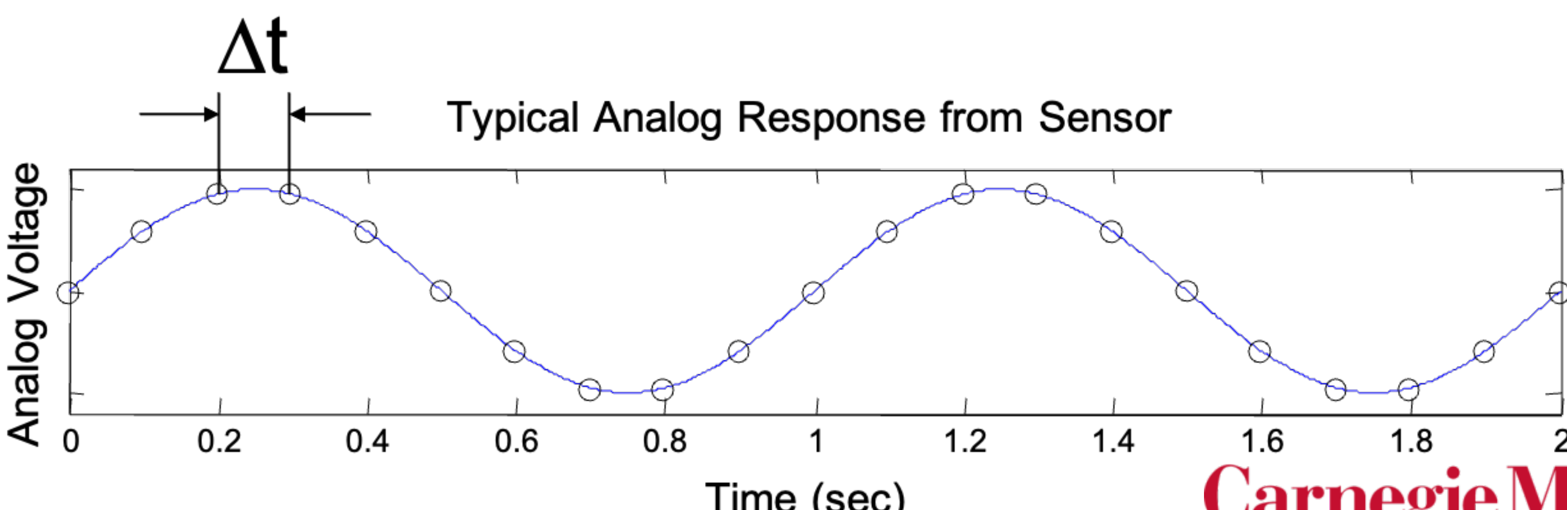
- Sampling frequency (Hz) is the inverse of the sampling interval.

$$f_s = 1/\Delta t \text{ (Hz)} \quad \text{or} \quad \omega_s = 2\pi/\Delta t \text{ (rad/sec)}$$

- Make sure the sampling frequency is at more than twice the desired frequency to be measured.

$$f_h < f_s/2 = 1/(2\Delta t)$$

- Sample 10 or more digital points during the signal period of interest.

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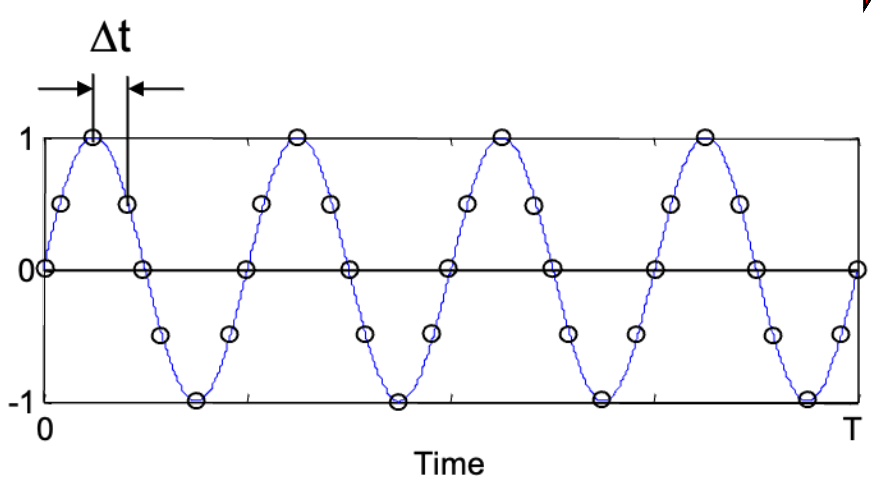
# Sampling Parameters

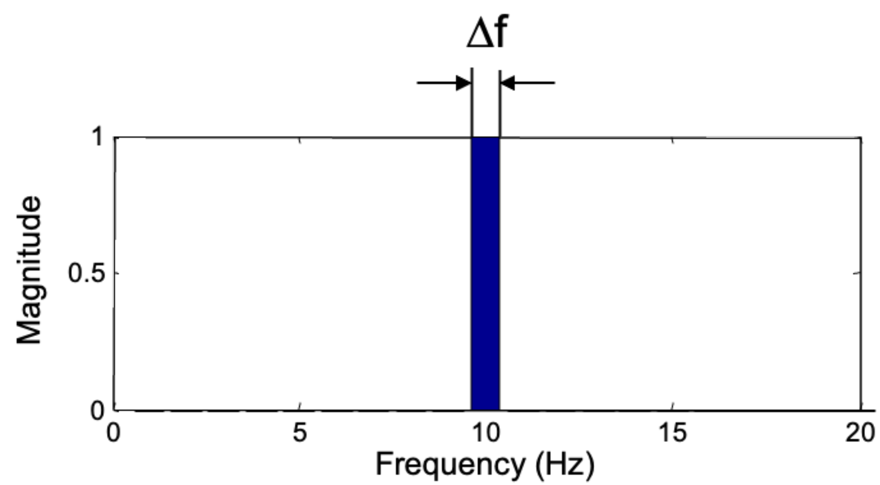
- Define the maximum frequency value to measure, then determine the sampling rate.

$$f_h < f_s/2 = 1/(2\Delta t) \Rightarrow \Delta t < 1/(2f_h)$$

- Define the desired frequency resolution, then select required number of data points.

$$\Delta f = 1/T = 1/(N\Delta t) \Rightarrow N = T/\Delta t = 1/(\Delta f\,\Delta t)$$

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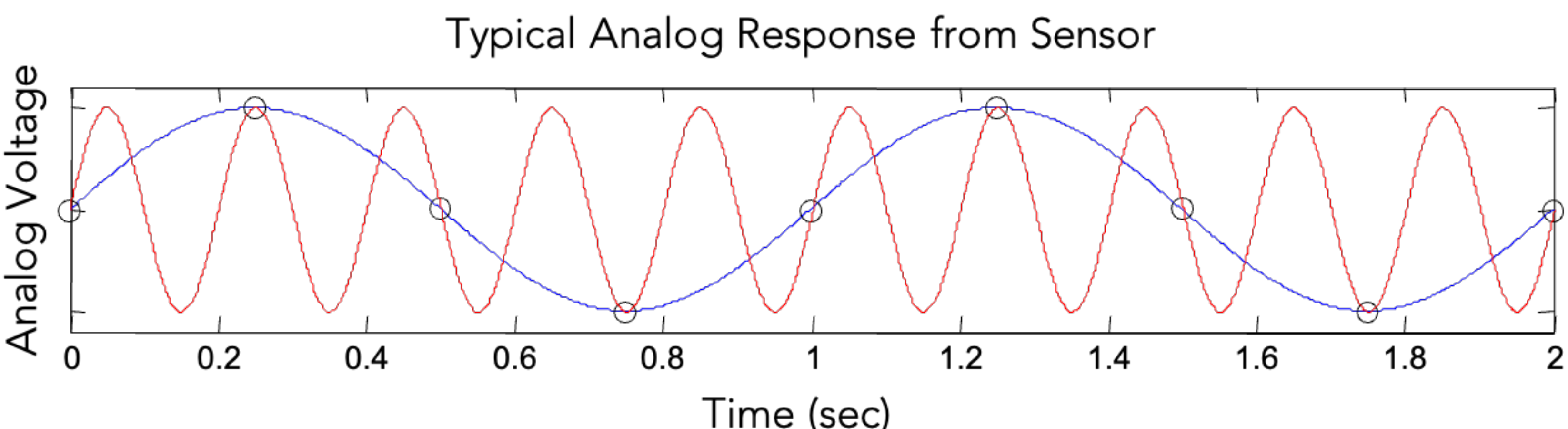
# Aliasing

$$y_1(t) = \sin(2\pi f_1 t) \qquad f_1 = 1\,\text{Hz}$$

$$f_S = 4\text{ Hz} \quad \text{or} \quad \Delta t = 0.25\text{ sec}$$

$$y_2(t) = \sin(2\pi f_2 t) \qquad f_2 = 5\,\text{Hz}$$

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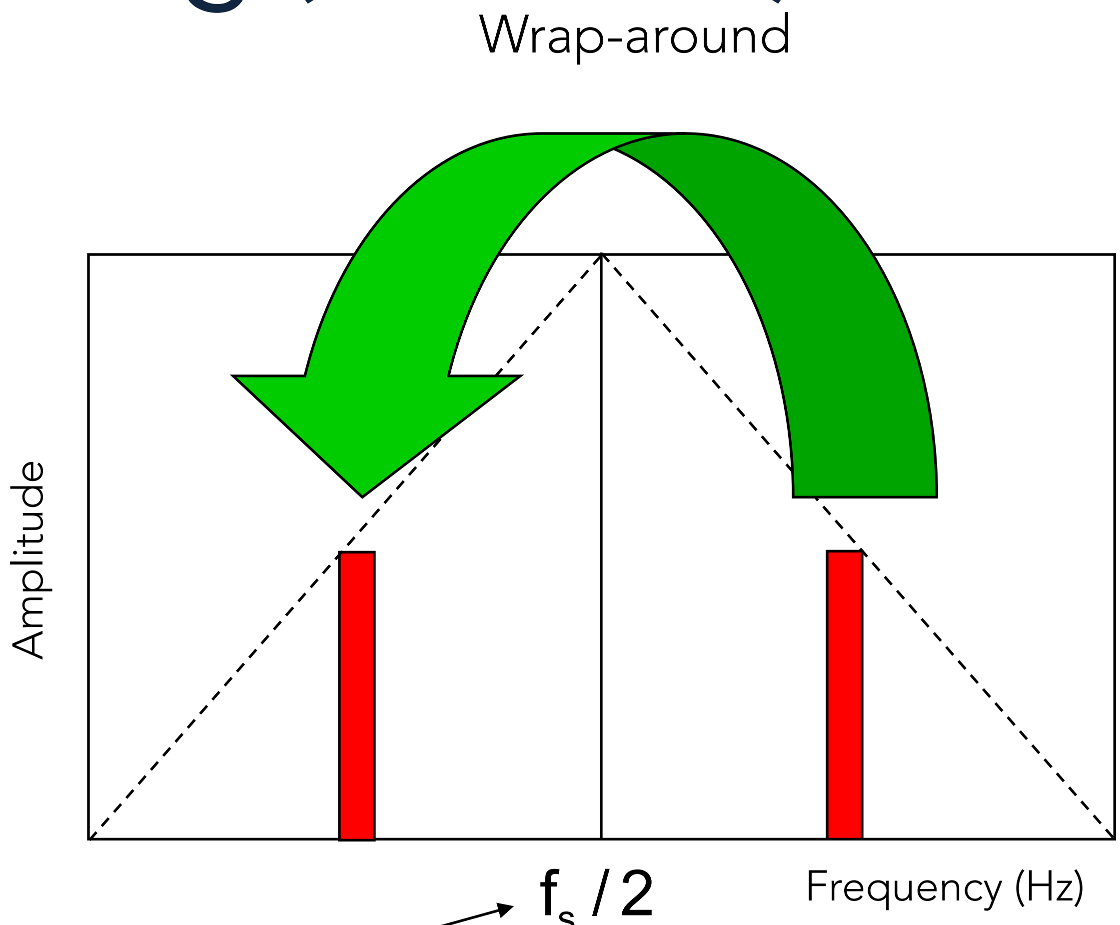
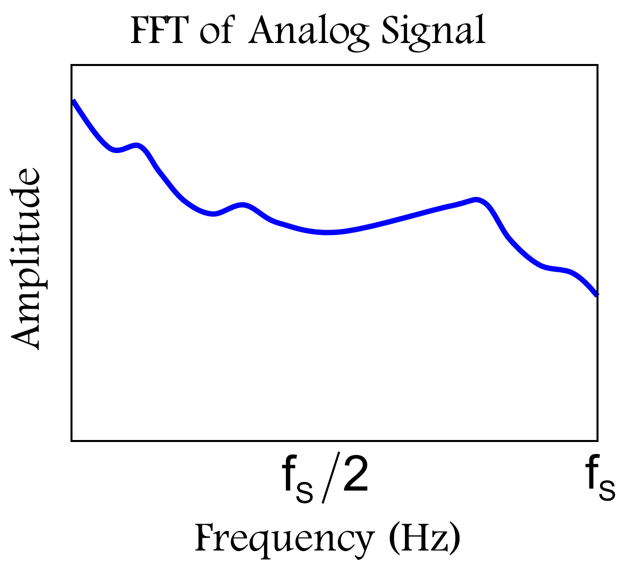
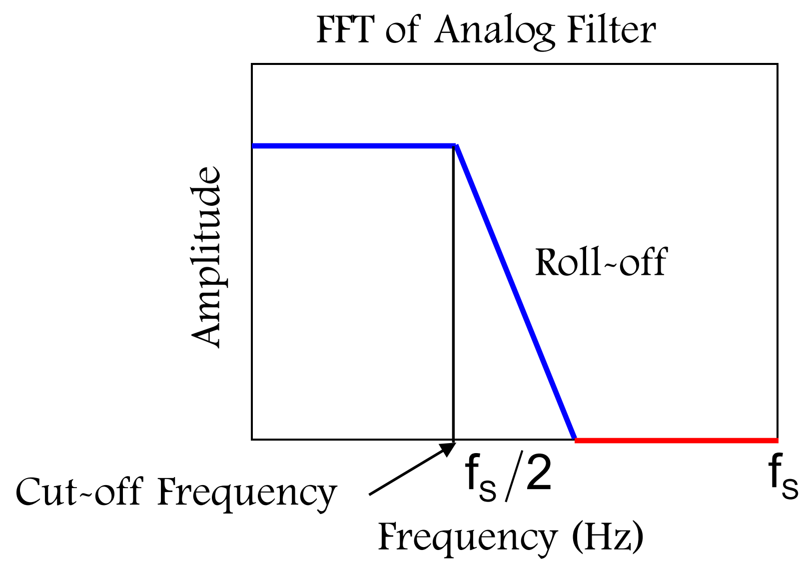
# Aliasing (continue)

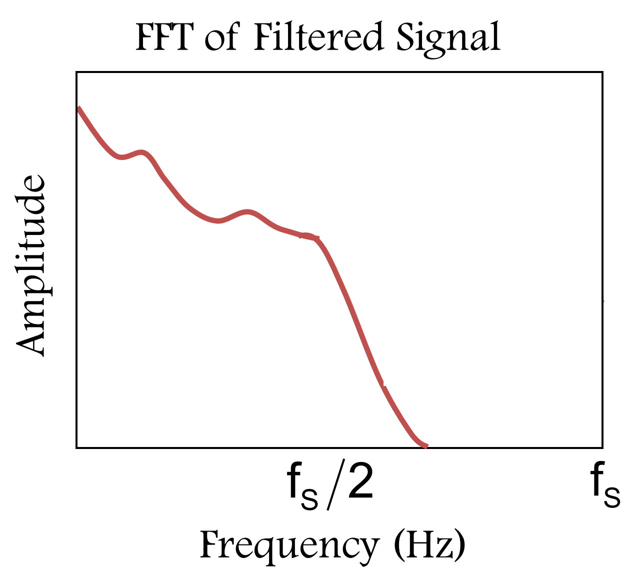



This aliasing occurs along the half of the sampling frequency.

# Anti-Aliasing Analog Filtering



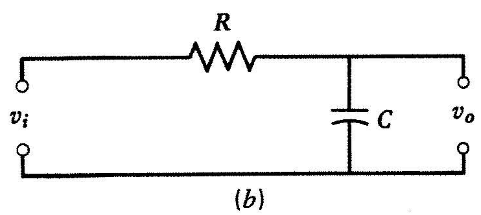



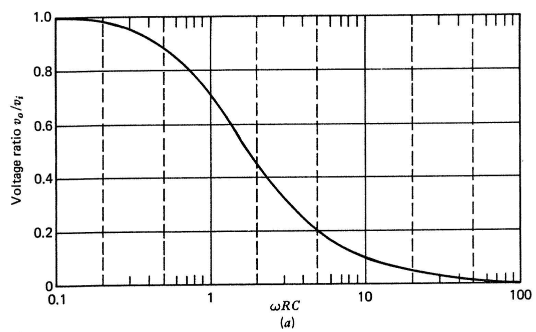




Anti-Aliasing is done before sampling.

# Analog Filter Design

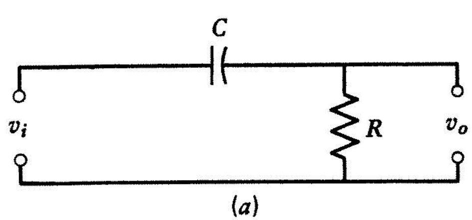
- Low-pass RC filter

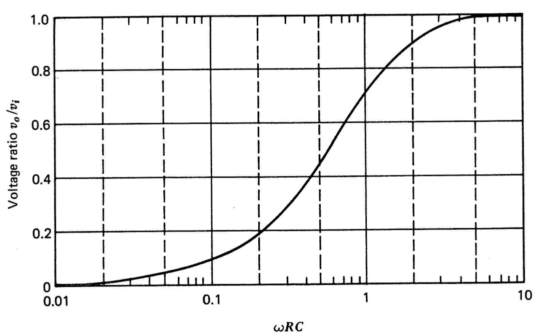



$$\frac{v_o}{v_i} = H(\omega) = \frac{1}{1 + j\omega RC} = |H(\omega)|e^{-j\phi}, \ |H(\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}} \quad \& \quad \phi = \tan^{-1}(\omega RC)$$

- High-pass RC filter

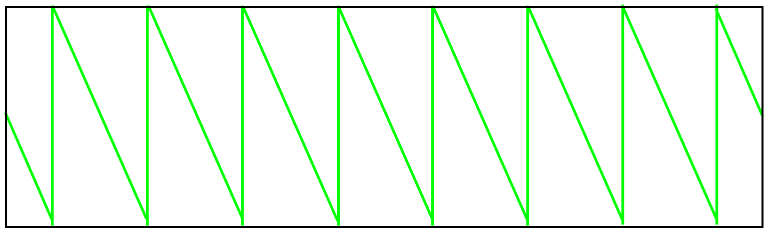



$$\frac{v_o}{v_i} = H(\omega) = \frac{j\omega RC}{1 + j\omega RC} = |H(\omega)|e^{j\phi}, \ |H(\omega)| = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}} \quad \& \quad \phi = \frac{\pi}{2} - \tan^{-1}(\omega RC)$$

# Quantization using 3-Bits Analog-to-Digital Converter

Quantization Error
(0.5 x Resolution)

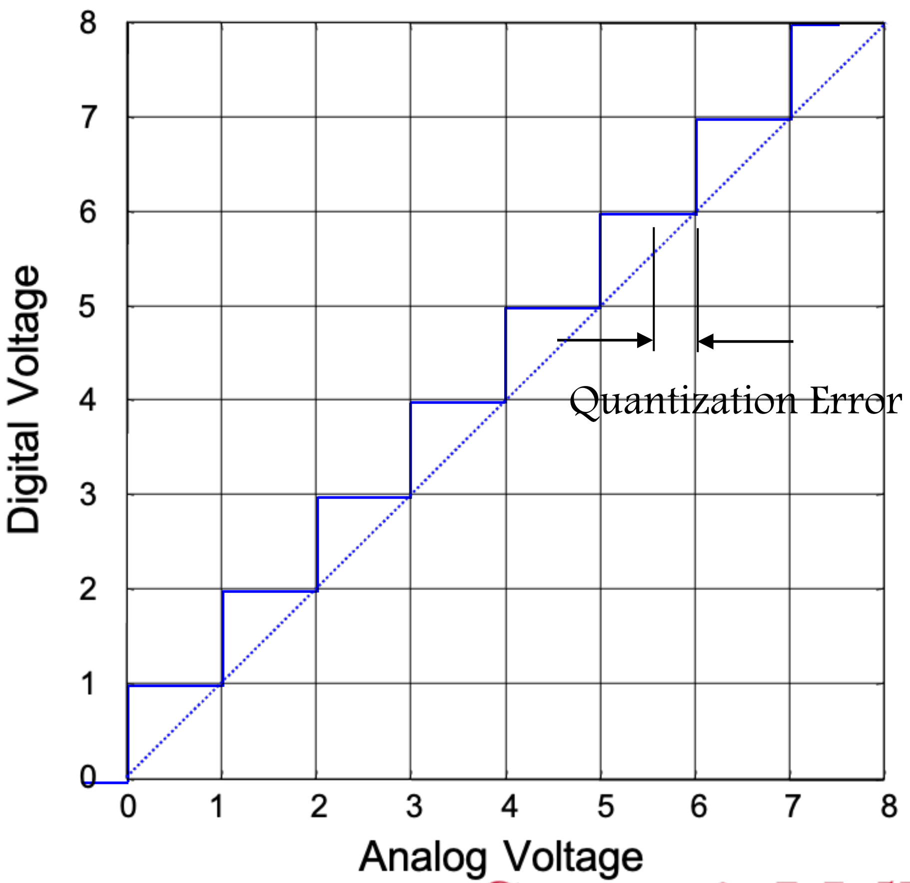


$$\text{Resolution} = \text{FSV}/2^3$$

3 bit A/D converter produces

$2^3 = 8$ bins

for full scale voltage (FSV)

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# Example for Resolution Calculation

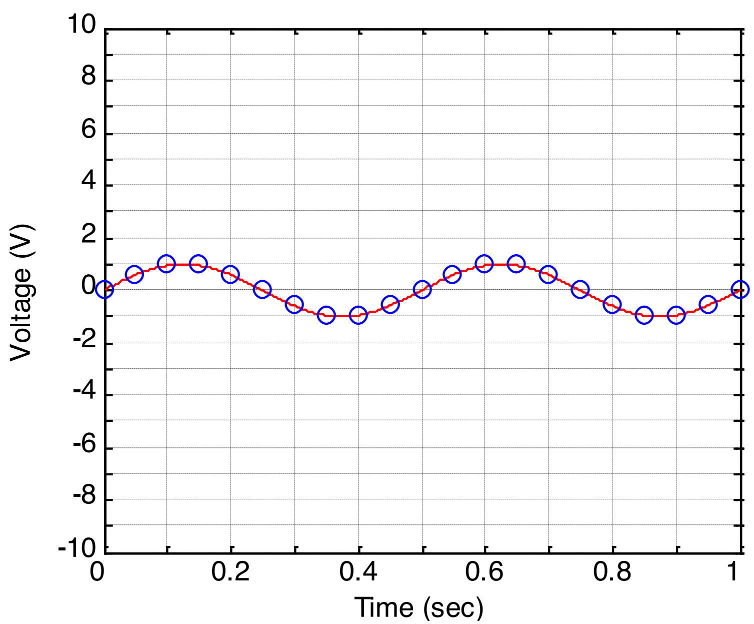
- Given parameters
  - Sensor sensitivity: 100 mV/g.
  - Voltage output range: - 2 V to + 2 V.
  - # of ADC bits: 16 bits.
- Calculation
  - 16 bit ADC divides the full scale voltage into $2^{16}$ = 65536 bins.
  - Resolution in terms of voltage: 4V/65536= 6.1035e-005 V.
  - Resolution in terms of acceleration:

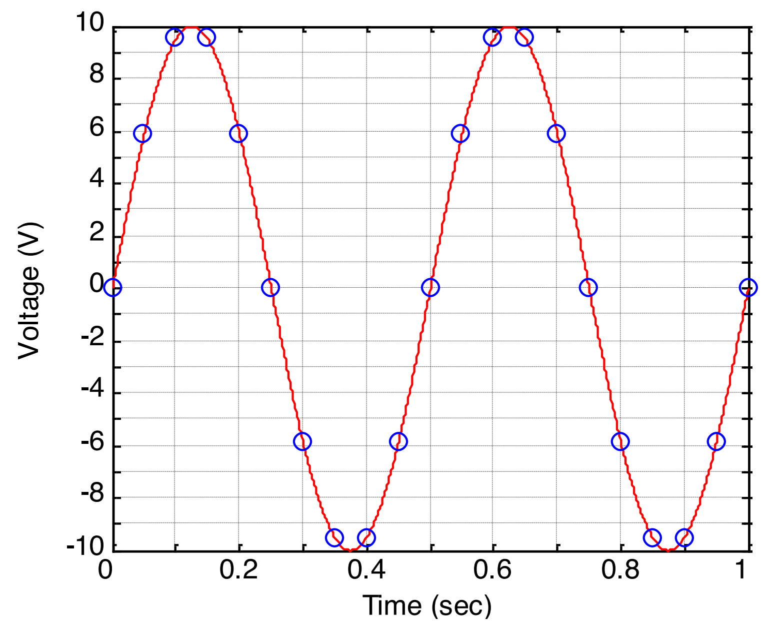
$$6.1035 \times 10^{-5}\,V \times \frac{1}{100mV/g} = 6.1035 \times 10^{-4}\,g$$

$$\text{Resolution}(g) = \frac{\text{Output voltage rage (V)}}{2^{\#\text{ of ADC bits}}} \times \frac{1}{\text{Sensor sensitivity (V/g)}}$$

# Other Considerations for Resolution

- Output Voltage Range: Keep the output voltage range small enough to prevent sensor signals from clipping (overload).
- Effective Resolution: When all of the available dynamic range of the ADC is not fully used, the effective resolution becomes worse. This also causes amplitude and phase distortion of the measured signals in both time and frequency domains.




Keep the maximum signal amplitude close to the dynamic voltage range.

The End

# QUESTIONS?

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