12-778 Fall 2022: Assignment #1

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Instructions

When answering the following questions, please provide all of your calculations to arrive at the answer (in addition to the answer itself). Your calculations should be very clear and easy to understand. They should include your assumptions, and a step-by-step explanation of how you arrived at the solution. Also, make sure you type your name and AndrewID somewhere on the first page, and that you clarify who you worked with in developing the intuitions behind your answer.

Some other recommendations:

- Before finding the answer to each question or looking at the next step in the solution, take some time to think about how you can come up with this on your own.
- Again, make sure you document everything you do, and not just write down the answer to the question. This will both help during grading as well as improving your learning process.
- Do not write down any solution or process that you do not understand. If you feel that you do not understand how to do something, seek some help. The preferred method for this is to post your questions on the discussion board for the course (i.e., Piazza).

What are sensors? (5%)

Pick an analog sensor from an online provider (one that you can easily purchase and would be interested in understanding better for your final project) and answer the following questions:

a) What are the potential applications for this sensor? Describe two.
b) How does this sensor operate, physically? In other words, how does the sensor transform energy in the physical phenomenon being measured into an electrical signal?
c) What is this sensor's accuracy? What is its transfer function? What other static and/or dynamic properties are worth discussing?

d) When considering the specific application that you have in mind, what are the advantages and disadvantages of this specific sensor compared to other alternative sensing technologies for the same physical stimulus?
e) How much does it cost, and where can you buy it?
f) What kind of interface circuit would be needed to connect it to your Raspberry Pi Pico W? Please sketch the circuit.

Working with sensors and your RPi Pico W (10%)

The ADC in your RPi Pico W has 3 available channels for you to supply whatever inputs you like, but there is a fourth channel that is directly connected to an on-board temperature sensor. You can easily access this fourth channel in the same way that you do the other three, but you won't need to connect any analgo sensor to it as it is hard-wired to the temperature sensor. Can you leverage the information on the Raspberry Pi Pico Python SDK (specifically, page 14) to interface with this sensor and answer the following questions?

a) What is the sensor's transfer function?
b) Why do we need to take the ADC measurements and divide them by 65535? Why do we multiply them by 3.3?
c) Create a Pico program that allows you to collect temperature data for a 10 minute interval, at 1s resolution.
d) Estimate the memory footprint of the data that you will collect using this program.
e) Collect data for 10 minutes and save it (either locally, or remotely). What is the file size? Compare it with the estimated memory footprint in the last question and comment on this.

Harmonic Oscillators (10%)

In class we discussed dynamic characteristics of sensors by looking into the response of single degree of freedom systems to harmonic loading. Unfortunately, we did not have enough time to solve the equations of motion for the damped forced oscillator case, or to play around with the resulting solutions. Thus, for this task, and to make sure the concepts are more intuitive to you, I ask that you play with a simulated harmonic oscillator, borrowed and later modified from here.

The problem is defined as finding the solutions to the following differential equation:

$$\ddot{x} + c\dot{x} + kx = \frac{F(t)}{m}$$

or, if we know that $\omega_0 = \sqrt{\frac{k}{m}}$, which is the angular frequency of the oscillator when undamped; and $\zeta = \frac{c}{2\sqrt{mk}}$ is the so-called damping ratio, we can rewrite it as such:

$$\ddot{x} + 2\zeta\omega_0\dot{x} + {\omega_0}^2 x = \frac{F(t)}{m}$$

We will start with the case where $\frac{F(t)}{m} = F_m sin(\omega_d t)$, i.e. the oscillator is driven by a sinusoidal force of amplitude F_m and frequency ω_d.

```
%matplotlib notebook
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy import integrate
import ipywidgets as ipw


def ode(X, t, zeta, omega0):
    """
    Free Harmonic Oscillator ODE
    """
    x, dotx = X
    ddotx = -2*zeta*omega0*dotx - omega0**2*x
    return [dotx, ddotx]

def odeDrive(X, t, zeta, omega0, omegad_omega0):
    """
    Driven Harmonic Oscillator ODE
    """
    x, dotx = X
    omegad = omegad_omega0 * omega0
    ddotx = -2*zeta*omega0*dotx - omega0**2*x + F_m * np.sin(omegad * t)
    return [dotx, ddotx]

def update(zeta = 0.05, omega0 = 2.*np.pi, omegad_omega0 = 1.):
    """
    Update function.
    """
    #zeta = c/(2.*np.sqrt(m*k))
    #omega0 = np.sqrt(k/m)
    #omegad_omega0 = omegad/omega0
    X0 = np.zeros(2)
    sol = integrate.odeint(odeDrive, X0, t, args = (zeta, omega0, omegad_omega0))
    line0.set_ydata(sol[:, 0])
```

```
    fig.canvas.draw()
    fig.canvas.flush_events()

Nt = 1000
F_m = 1.
t = np.linspace(0., 10., Nt)
dummy = np.zeros_like(t)
fig = plt.figure()
line0, = plt.plot(t, dummy, label = "position")
plt.grid()
plt.ylim(-1., 1.)
plt.xlabel("Time, $t$")
plt.ylabel("Amplitude, $a$")
plt.legend()

ipw.interact(update, zeta = (0., .2, 0.01),
             omega0 = (2.*np.pi*0.5, 2.*np.pi*5, 2.*np.pi*0.01),
             omegad_omega0 = (0.1, 2., 0.05));
```

```
<IPython.core.display.Javascript object>
```

```
<IPython.core.display.HTML object>
```

```
interactive(children=(FloatSlider(value=0.05, description='zeta', max=0.2, step=0.01), FloatS
```

To start, take the code above and copy/paste it to a Jupyter Notebook so that you can play around with the interactive interface that it provides. Currently, the interface is set up to allow you to set three parameters, namely ζ, ω_0 and $\frac{\omega_d}{\omega_0}$. That said, it is relatively easy to change the interface so that you can directly alter c, m, k and ω_d. Your task is to leverage the interactive interface and get familiarized with the response of the system as you change the damping, stiffness, mass and frequency of the harmonic loading. Try answering these questions for yourself:

a) What is the relationship between the amplitude and frequency of the harmonic loading, and the amplitude and frequency of the system's response?
b) What happens when you drive the system at the resonant frequency?

Once you are done, please write a brief summary of your overall findings (2 or 3 paragraphs of thoughts, not just about the last few questions) as your answer to this part of the assignment.

Circuit Analysis (25%)

Task a (10%)

If you think carefully about what we've learned about complex impedances and how they work in AC circuits, you'll quickly realize that almost any component in your system can be seen as a filter. It may not be evident, but even the wire you are using to connect components together, itself, can act as a filter. Let's explore that a bit.

Virtually all cables have a very small, but detectable capacitance. This is because the insulation material around each of the wires closing the circuit acts as a dielectric and can accumulate charge when a voltage is present.

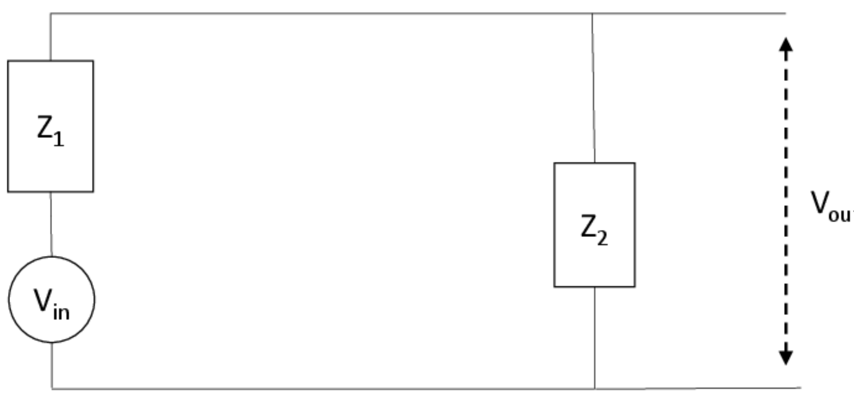

Figure 1: Simple circuit generalizing the effect of cable resistance and capacitance.

Figure 1 shows a diagram of how this works out in practice. If, for example, we had an ideal microphone as the voltage source (V_{in}) in this circuit, and wanted to measure the signal coming out (V_{out}), we would find that the values of Z_1 and Z_2, namely the resistance and capacitance of the cable itself, would influence the signal we receive.

To study the effect of this filter, let's analyze the ratio of the magnitudes for V_{in} and V_{out}. In other words, let's study how the voltage we measure is related to the voltage being supplied by the microphone, as described below:

$$\frac{V_{out}}{V_{in}}$$

Task a: If $Z_1 = R$ and $Z_2 = \frac{1}{j\omega C}$, where C is the capacitance of the cable, and R is the resistance, then what is the expression for $\frac{V_{out}}{V_{in}}$?

Task b: The capacitance of the wire (as well as the resistance) increase with its length l. In other words, $C \propto l$ and $R \propto l$. What will happen to the dynamic properties of the measured signal from a microphone as we increase the length of the wire?

Task b (5%)

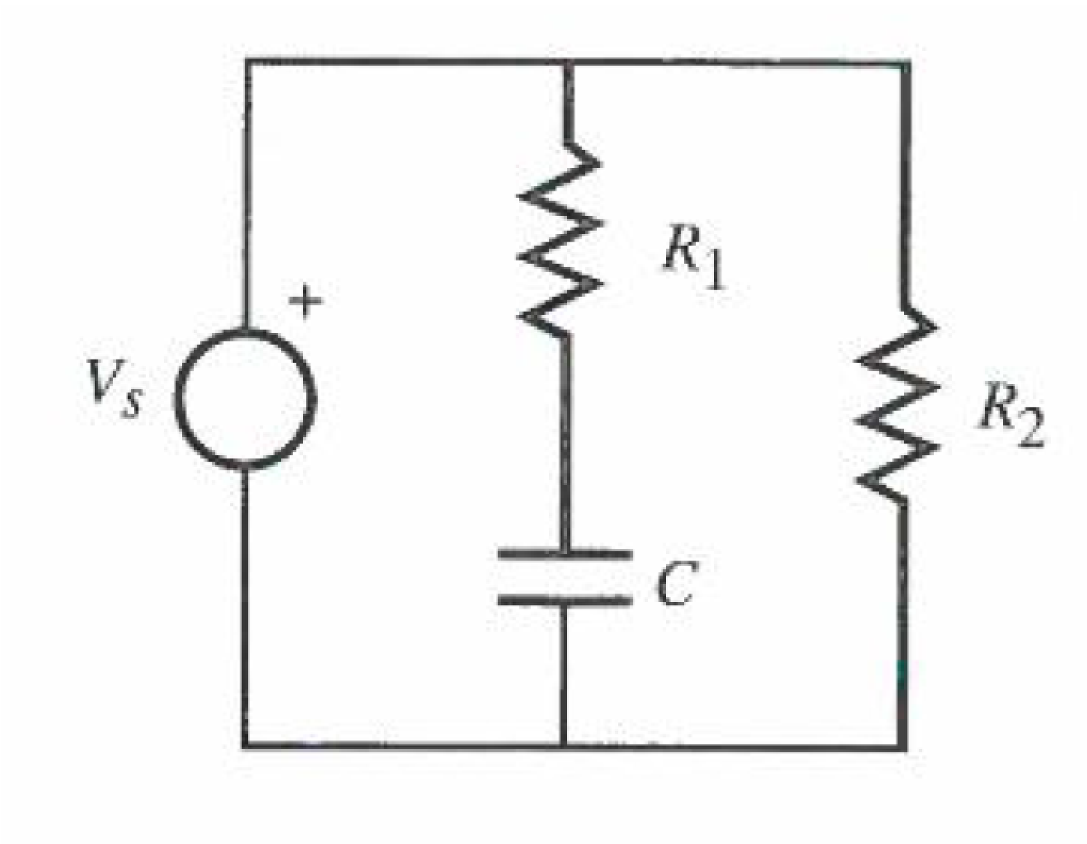

Figure 2: Simple DC circuit

For the circuit in Figure 2, find the steady-state voltage across R_1, R_2 and C, if $V_s = 10$ V DC, $R_1 = 1$ kΩ, $R_2 = 1$ kΩ and $C = 0.01\mu$F.

Task c (10%)

Solve Exercise 2.24 from Chapter 2 (Analyis of Circuits) from Instrumentation for Engineering Measurements by Dally, Riley and McConnell.

Impedance (20%)

In chapter 2 of Fraden, Figure Figure 3 shows up (Figure 2.15 in the book):

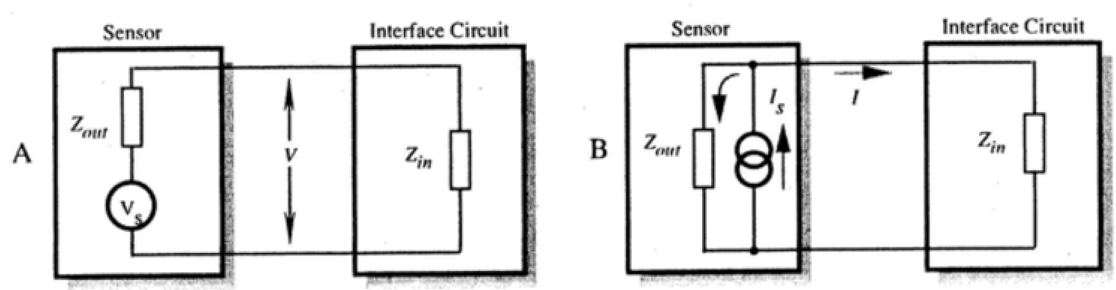

Figure 3: Sensor connections to an interface circuit. (a) shows a sensor with voltage output, while (b) shows a sensor with current output.

Task a: The concept of impedance is very important for circuit analysis. Please define, as best as you can (i.e., using technical concepts and mathematical notation), what impedance is.

Task b: Impedance matching is also an important concept, as described by Fraden in page 40 of Chapter 2. Can you explain why it is recommended that a sensor with voltage output should have a much smaller impedance (Z_{out}) compared to the impedance (Z_{in}) of the interface circuit? (see Figure 3 A). Why is it different for Figure 3 B?

Task c: Find a good video online explaining the concept of impedance matching (one whose explanation you find intuitive and clear) and provide the URL to that video as the answer to this question.

Task d: Compute the total impedance for the AC circuit shown in Figure Figure 4. Here, v_s is an AC voltage source (i.e., it is a time-varying source of voltage, varying as follows: $v_s = v_i e^{j\omega t}$). The frequency of this AC source is ω. In the same circuit, R, L and C represent the resistance, inductance and capacitance values for those circuit elements, respectively.

Analog-to-Digital Conversion (10%)

Suppose I set out to collect measurements about the voltage supplied by the electrical utility company to my house house for a week. I happen to know that the frequency of this voltage (in the US) is somehwere around 60Hz, but given that there is no guarantee it will maintain this frequency, and also considering the fact that the signal is not band-limited, I decide to over-sample.

Task a: Suppose I decide to sample it at 12 kHz with a 12-bit ADC. If I collect measurements for an entire week, how much memory will I need to store all of these samples?

Task b: Suppose now that I figure out a way to effectively make the signal band-limited, and I can guarantee that all the signal content will be below 70Hz. What would be a more efficient sampling rate in this case? How much memory would I require in this case?

Aliasing (10%)

In class, we learned why aliasing occurs and how it is related to the sampling frequency (or the Nyquist frequency) of the data acquisition configuration. Answer the following questions related to aliasing:

Task a: A 10Hz pure sine wave is sampled at 12 Hz. Compute the maximum frequency that can be represented in the resulting discrete signal. Compute the aliased frequency.

Task b: Assume that the measured signal is complex periodic of the form $y(t) = A_1 sin(2\pi 25t) + A_2 sin(2\pi 75t) + A_3 sin(2\pi 125t)$. If this signal is sampled at 100Hz, determine the frequency content of the resulting discrete response signal.

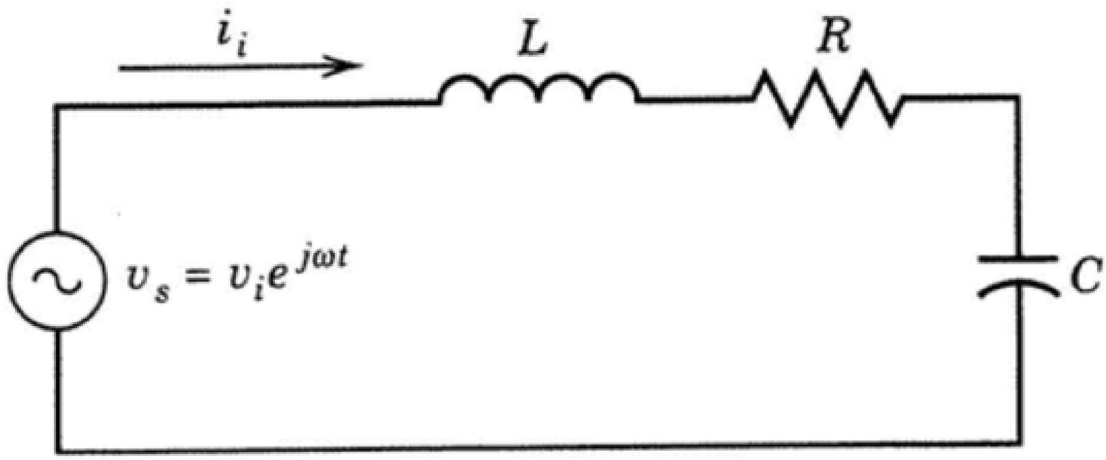

Figure 4: An AC circuit containing a resistance, capacitance and inductance in series.

Filters (10%)

A moving average is an filtering technique that can be applied to an analog or digital signal. A moving average is based on the concept of windowing as illustrated in Figure 5. The portion of the signal that lies inside the window in averaged and the average values are plotted as a function of time as the window moves across the signal. A 10-point moving average of the signal is plotted as well in Figure 6.

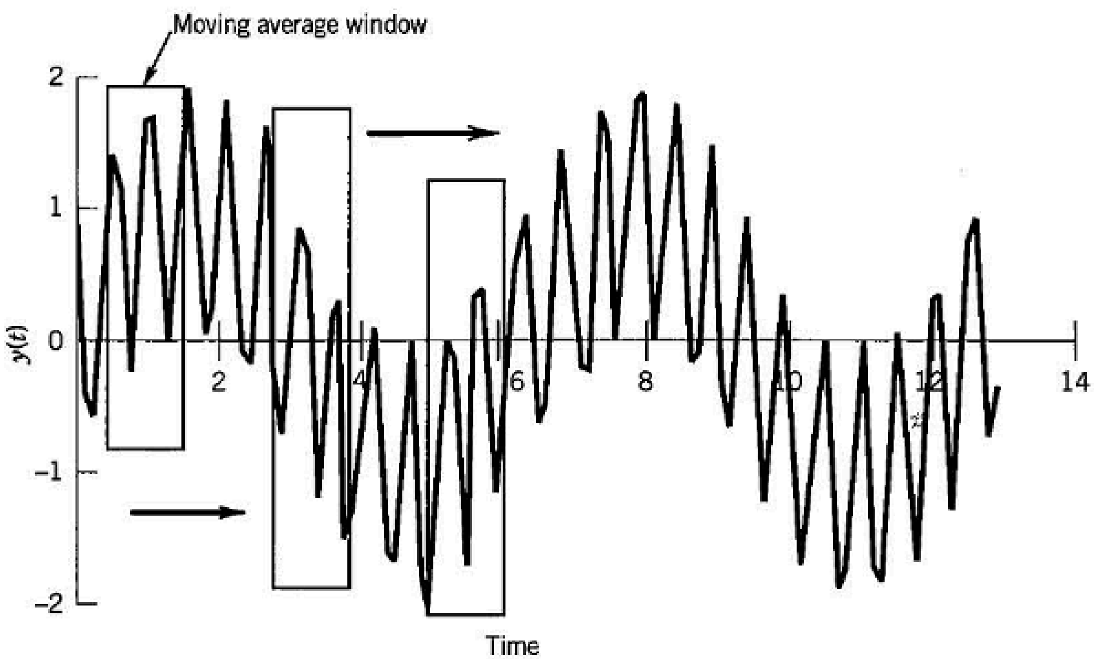

Figure 5: Moving averaging and windowing

Task a: Discuss the effects of employing a moving average on the signal depicted in Figure 5. In particular, discuss the changes imparted to the dynamic characteristics of the signal. What does this say about the the transfer function for the moving average filter?

Task b: Develop a simple Python program that computes the moving average for the following signal: $y(t) = sin(5t)+cos(11t)$, discretized by applying a 0.05second sampling train. Examine the effects of chaning the averaging window size from 4 to 30 samples.

Task c: What did you learn about the effects of the widht (number of samples) for the averaging window?

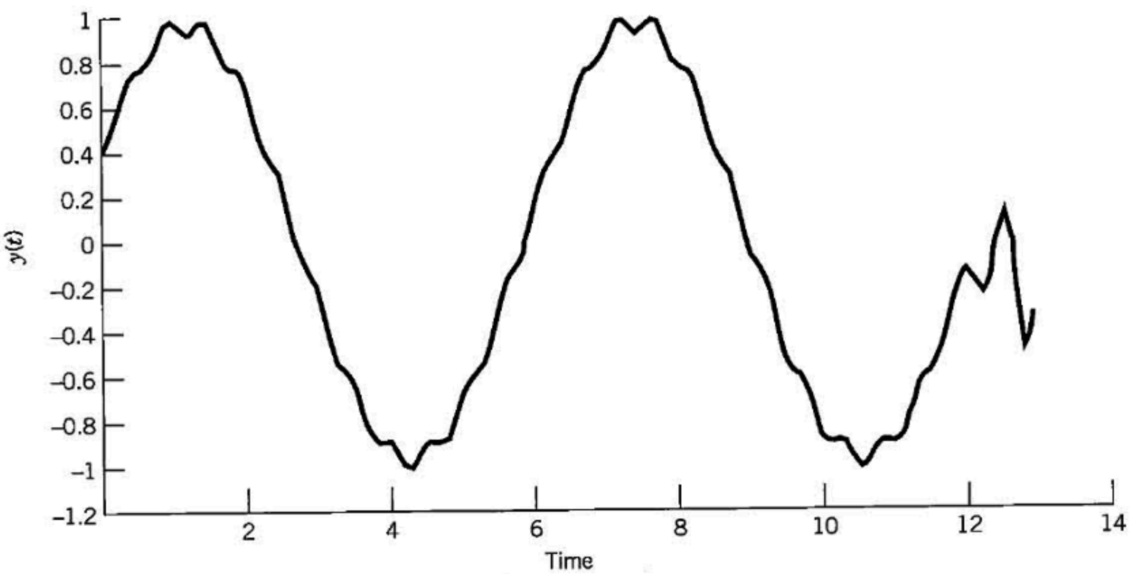

Figure 6: Effect of moving average on the signal.